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The many-worlds and relative states interpretations of quantum mechanics, and the quantum Zeno paradox

D Home† and M A B Whitaker
Department of Physics, University of Ulster, Coleraine BT52 1SA, UK

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Abstract. The quantum Zeno paradox is examined within the many-worlds and relative states interpretations of quantum mechanics. In the many-worlds interpretation the effect is predicted to persist. The possibility of recombining worlds is not expected to be relevant. In the simplest form of relative states interpretation the effect may be avoided but this form of interpretation experiences difficulties in coping with conventional problems of quantum theory. The more complex version of the relative states interpretation, which takes account of the correlations of system with apparatus, predicts the occurrence of the paradox.

1. Introduction

The quantum Zeno paradox has come into prominence rather later than the other celebrated paradoxes of quantum theory (EPR, Schrödinger's cat), but there have recently been a number of discussions from different points of view (Chiu *et al* 1977, Misra and Sudarshan 1977, Peres 1980a, Singh and Whitaker 1982, Peres 1984, Home and Whitaker 1986 (hereafter referred to as I)). (We use the word 'paradox', incidentally, in a non-controversial sense.)

In I we pointed out that the Zeno paradox is a paradox of prediction; it is the predicted results of conventional interpretations of quantum theory that appear strange. The EPR and cat paradoxes, in contrast, are paradoxes of interpretation; the results of the thought experiments are not objectionable in themselves, but analysis of them in terms of the usual interpretations of quantum mechanics presents difficulties. The Zeno paradox, then, may be experimentally tested. If different interpretations of quantum mechanics give rise to different predictions as to this particular paradox, it opens up the possibility of being able, at least in principle, to check which interpretations need to be considered.

Among those interpretations of quantum theory alternative to that of von Neumann (1955) are the ensemble and many-worlds interpretations, both of which claim to eliminate the wavepacket collapse responsible for the difficulties of the von Neumann scheme. In I we showed that the quantum Zeno paradox persists within an ensemble interpretation. In contrast, the EPR and cat paradoxes are usually claimed to disappear within this interpretation (see, for example, Ballentine 1970). Thus the Zeno paradox cannot be used to differentiate between the von Neumann and ensemble interpretations.

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† Permanent address: Theoretical Physics Division, Saha Institute of Nuclear Physics, Calcutta 700 009, India.

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Let us briefly describe the argument used in I to establish this point. We work with the density matrix, the natural language to be used in handling an ensemble interpretation. We use a representation in which the dynamical observable being measured is diagonal. Before the measurement, the density matrix will, in general, be non-diagonal, but may be taken as pure (if necessary by restricting oneself to a particular sub-ensemble). At the measurement, the diagonal elements are unchanged and this enables the EPR and Schrödinger's cat paradoxes to be handled by the formalism. However the off-diagonal elements disappear and the density matrix becomes mixed. As shown in I, this is sufficient to cause a quantum Zeno paradox.

It is natural to proceed to analyse the same situation with respect to the many-worlds interpretation. The claim made for this interpretation is that the wavefunction changes only in accordance with the Schrödinger equation and no wavefunction collapse takes place. This would suggest that the quantum Zeno paradox may be avoided in such an interpretation.

In a previous paper (Whitaker 1985a, hereafter referred to as II), the opinion was expressed that, among the pioneering papers setting out the many-worlds interpretation, there are, in fact, two rather distinct sets of ideas put forward.

The first will be called here, as in II, following Everett (1957a), the relative states interpretation (RSI). The clearest account of this interpretation is in the paper by Cooper and van Vechten (1969). The papers by Everett (1957a, b (reprinted as Everett 1973)), Zeh (1970) and, among more recent discussions, Page (1982) appear to be close in spirit to this approach.

The other strand of interpretation is that started by DeWitt (1968, 1970). It is to this interpretation that we limited the term many-worlds interpretation (MWI) in II and we do the same here. It was suggested in II that the work of Kunstatter and Trainor (1984) was close to this approach.

To explain the differences between the RSI and the MWI, let us describe briefly the general measurement problem. Let us consider a system which has a dynamic observable O with eigenfunctions $\phi_1, \phi_2, \ldots,$ and corresponding eigenvalues $O_1, O_2, \ldots.$ Before a measurement of O, let us say that the wavefunction is given by $2^{-1/2}(\phi_1 + \phi_2)$. The usual analysis following von Neumann (1955) tells us that there are equal probabilities of the values O_1 and O_2 being found in a measurement of O and that, if the value O_m is found, the wavefunction collapses to ϕ_m.

In contrast the RSI denies that any wavefunction collapse takes place. The problem, of course, is how to explain the fact that a particular value, O_m, is found. Cooper and van Vechten (1969) attempt to solve this problem by putting the mind of the observer inside the realm of the Schrödinger equation. The total wavefunction of observed system, observing apparatus and mind remains uncollapsed; the mind itself, though, is in a particular state and holds a particular value O_m. Other proponents of the RSI describe the situation in other terms, sometimes similar, often less precise. Everett (1957a, b), for example, places much more stress on consistency within a given world component (that is to say, using one component of the wavefunction only following a measurement) than on the basic question of why one uses only one component in the first place.

Again, in contrast, the MWI explains clearly exactly what transpires at the measurement. The wavefunction does not collapse, ϕ_1 and ϕ_2 both exist after the measurement, but in different worlds. The value O_m is, of course, accepted in the world in which the wavefunction after measurement is ϕ_m. The 'other world' (or, in general, 'worlds') are in no sense, it seems, to be considered as some sort of mathematical fiction.

In II it was suggested that the MWI and the RSI face complementary problems. Given its starting position, the RSI is able to discuss the problem of measurement without logical difficulty, but it is this very starting position (in particular, its unwillingness to discuss the 'other components') that renders it unconvincing to the majority of physicists. In contrast, the general position of the MWI is extremely clear; it is suggested in II, though, that it does not, in fact, solve the problems left by the von Neumann formalism. The question of what constitutes an 'observation' and what merely a disturbance, the status and function of 'the observer' seem as thorny questions for DeWitt as for von Neumann. In II, the EPR experiment was analysed within the RSI and the MWI, and the conclusions reinforced the general discussion of measurement just outlined. In the following section we discuss how the MWI and a number of forms of the RSI discuss the quantum Zeno paradox.

2. The quantum Zeno paradox and the many-worlds and relative states interpretations of quantum mechanics

It is convenient to consider the MWI first. Suppose the wavefunction of the decaying nucleus at the time of the first measurement is $\alpha_s\phi_s+\alpha_d\phi_d$, where ϕ_s and ϕ_d correspond to surviving and decayed systems, respectively, and α_s and α_d are coefficients with $|\alpha_s|^2+|\alpha_d|^2=1$.

At the time of the measurement we must suppose that a number of worlds, N_s+N_d, are created, in N_s of which there is a surviving nucleus, and in N_d of which there is a decayed nucleus. We must have $N_s/N_d=|\alpha_s|^2/|\alpha_d|^2$, and presumably N_s and N_d will have no common factor. (We ignore the complication caused if $|\alpha_s|^2$ and $|\alpha_d|^2$ are irrational, a complication that is potentially present in any world splitting.) In any of the N_s worlds in which the nucleus survives, during the splitting the part of the wavefunction containing ϕ_s is 'cut off from' that containing ϕ_d. As shown in I, in density-matrix terms, the change is from an idempotent matrix with non-zero diagonal and off-diagonal terms to a diagonal non-idempotent matrix. (For our purposes we may use 2×2 matrices, the first state being for surviving systems, the second for decayed systems.) This corresponds to a change from pure to mixed wavefunction. Since the off-diagonal terms in the density matrix are responsible for further decay, the stripping-off of the off-diagonal terms, or the change from pure to mixed wavefunction, inhibits further decay or, in the limit of continuous measurement, prohibits it. The quantum Zeno paradox persists, therefore, within the MWI.

As in II we find the predictions of the MWI parallel those of the von Neumann approach. This is not surprising; the MWI is set up specifically so that the experience of any observer duplicates that of a corresponding operator in the von Neumann formalism. As in II, we find the MWI perfectly clear in its conception, but question its ability to solve problems left unsolved by more usual interpretations.

Since it has been said that the persistence of the quantum Zeno effect is a result of splitting worlds, it is worth analysing whether the so-called recombination of worlds, often mentioned though seldom discussed in the literature, would change the situation. The term has, in fact, been used in (at least) two different ways. First, it may be said that the results of observations in any particular world form a series of data sets in a memory of limited size. Following a measurement, two worlds may be split because of differences in an initial piece of data, but agree on subsequent pieces. When the mismatching data have to be jettisoned because of lack of space, the worlds become identical and may be said to have recombined. This argument appears to have little relevance to the present discussion.

A second use of the idea of recombination of worlds occurs in a reply by Clarke (1976) to a note by Kerr (1976), itself commenting on an account of the MWI by Clarke (1974). The discussion concerns a Stern-Gerlach apparatus, with inhomogeneous field in the x direction, having incident on it a beam of atoms with S_z equal to $\frac{1}{2}$, and producing two beams of atoms with S_x equal to $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. Kerr suggests that these states are macroscopically distinguishable, and that the Stern-Gerlach experiment is therefore performing a measurement and, in an MWI, splitting worlds. It is well known, however, that the beams may, at least in principle, be recombined to produce the original beam (i.e. a pure state, not a mixture of particles with S_x equal to $+\frac{1}{2}$ and $-\frac{1}{2}$). If the beam splitting is to be regarded as a world splitting, then Clarke (1976) suggests that the recombination of beams may be regarded as a recombination of worlds.

The approach taken here, along the lines of one which Kerr (1976) suggests, but apparently does not accept, is that no measurement is implied by the initial beam splitting (see, for example, Dicke and Wittke 1960, Singh and Whitaker 1982). In a measurement process, it is suggested, the initial correlating of system and measuring apparatus states assumes a highly simplified apparatus with a limited number of degrees of freedom, and this stage is reversible. It is followed (Peres 1980b) by an irreversible process in which the remaining degrees of freedom of the apparatus take part, and it is only at this stage that a permanent mark may be left and a measurement deemed to have taken place. In the Stern-Gerlach case, the second stage does not take place and it seems unnecessary to describe the beam splitting as a measurement. There is therefore no world splitting and therefore no requirement for a subsequent recombination.

It does not appear, then, that the concept of recombining worlds affects the argument of this section.

When we turn to the RSI, we need to consider explicitly versions which differ in their attitude to that part of the wavefunction which may be ignored by the observer. If one believes simply that the wavefunction contains both ϕ_s and ϕ_d components, and is thus completely unchanged by the observation, there is clearly no inhibition of further decay caused by separation of the two terms, and hence no quantum Zeno paradox. The mind, of course, contains only one of the results, decay or survival, with relative probabilities of $|\alpha_d|^2$ and $|\alpha_s|^2$.

Such an approach to measurement may be said to follow directly from the initial precept of the RSI—the wavefunction changes only according to the Schrödinger equation. It is satisfactory where the operator $\hat{O}$ for the dynamic observable being measured, O, commutes with the Hamiltonian, $\hat{\mathscr{H}}$. The wavefunction will evolve as

$$2^{-1/2}[\phi_1 \exp(-\mathrm{i}E_1 t/\hbar) + \phi_2 \exp(-\mathrm{i}E_2 t/\hbar)]$$

following the measurement, where the ϕ_m, as before, are eigenfunctions of $\hat{O}$ with eigenvalues O_m. If the result of the first measurement were O_m, only the mth component of the wavefunction need be taken into consideration according to the RSI, and the result of a second measurement at a time t later than the first will give the same answer, as for the von Neumann interpretation.

However if $\hat{\mathscr{H}}$ and $\hat{O}$ do not commute, ϕ_m will, of course, evolve not as $\phi_m \exp(-\mathrm{i}E_m t/\hbar)$ but as $\Sigma_n\, c_{mn}\beta_n \exp(-\mathrm{i}E_n t/\hbar)$, where the β_n are eigenfunctions of $\hat{\mathscr{H}}$ with eigenvalues E_n. The functions evolving from ϕ_1 and ϕ_2 may be expected to interfere drastically with each other. In particular, if ϕ_m evolves into $\phi_m(t)$, then $\phi^*_{m_1}(t)\phi_{m_2}(m_1 \neq m_2)$, integrated over all space, will not, in general, be zero. Put simply, ϕ_2 may evolve into ϕ_1. It may seem extremely artificial to maintain the claim that one is retaining the entire wavefunction if one does not take this interference seriously. Yet if one does include it one will not retain the results of the von Neumann approach. This goes against the declaration of Everett (1957b) that the aim of his approach is 'not to deny or contradict the conventional formulation of quantum theory... but rather to supply a new, more general and complete formulation, from which the conventional formulation can be *deduced*'.

Even if this demand is relaxed, this simple approach to the RSI still faces difficulties. Not surprisingly it appears to fail on precisely those types of problem the collapse of the wavefunction is designed to solve. For example, it does not appear to give the result that two successive measurements of the position of a particle, separated by a very short time, should give results very close together.

What we believe is a more satisfactory approach using the RSI recognises the correlation between system and observing apparatus following an observation. We use the terms of Cooper and van Vechten (1969), who give the most explicit discussion of this point, though the analysis is applicable to all versions of the RSI, including that of Everett. The analysis does not allow interference between the two components of the wavefunction following a measurement. If the general state of the apparatus is written as A, a typical measurement scheme may be written as

$$2^{-1/2}(\phi_1 + \phi_2);\ A \to 2^{-1/2}(\phi_1 A_1 + \phi_2 A_2). \tag{1}$$

A_1 and A_2 are the states of the apparatus corresponding to eigenvalues O_1 and O_2 of operator O. The transition represented by (1) is necessarily irreversible, i.e. the definition of a genuine measurement, indeed. The states A_1 and A_2 may be registered, for example, by permanent marks on paper. It is not possible for the state of the apparatus, then, to change from A_1 to A_2, or vice versa, and this means that there can be no interference between the two components on the right-hand side of (1). From the point of view of the observer, one of the two components may be ignored.

The measurement process in a Zeno decay (of, say, a radioactive nucleus) obeys the prescription just given. Although ϕ_s may develop into ϕ_d, the corresponding apparatus state, A_s, which may be represented by a blank strip of paper for a particular time interval, cannot develop into A_d, the same strip of paper, for the same time interval, with a darkened region. According to Cooper and van Vechten, then, ϕ_s and ϕ_d are essentially separated and cannot interfere; hence a quantum Zeno paradox *is* predicted.

Since we have mentioned various versions of the RSI, it may be appropriate to mention a paper by Page and Geilker (1981) which allows very much greater effectiveness to the 'other' components. Their suggestion is in regard to quantum gravity, but it has been claimed that their hypothesis would lead to results out of accord with experience (Whitaker 1985b).

3. Discussion

Earlier in this paper it was pointed out that the existence or otherwise of the quantum Zeno effect would give evidence as to which interpretations of quantum mechanics need to be considered. Since, in fact, most novel interpretations of quantum mechanics are explicitly designed to agree with, rather than contradict, more established interpretations as regards physical predictions, it is not surprising that nearly all the interpretations we have considered, here and in I, agree with the existence of the quantum Zeno effect.

If the paradox is observed, then, not much information will have been obtained. As discussed in I, there are indeed extremely tentative suggestions that the effect is demonstrated by the failure to observe the proton decay predicted by the grand unified theories (GUT).

It would be particularly intriguing, however, if it became clear that the quantum Zeno effect did not take place. The only hint of an interpretation predicting such a negative result is the simplest form of RSI which, since it allows, in all cases, interference between components of the wavefunction corresponding to different observations, experiences problems elsewhere. We may recall from II that the RSI was relatively successful, within its own terms of reference, in dealing with the EPR problem. Non-existence of the Zeno effect could be a call for the work of Everett and his followers to be analysed much more thoroughly than so far, to see if a theory can be produced which avoids the difficulties of the simpler forms of RSI but retains the positive features.

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